18010

B.C.A. Examination, June-2023

MATHEMATICS-II

(BCA-201)

Time : 3 Hours] *[Maximum Marks : 75*

Note : Attempt **all** the sections as per instructions.

Section–A

(Very Short Answer Questions)

Note : Attempt all the *five* questions. Each question carries 3 marks. $5 \times 3 = 15$

1. Define cardinality of set with suitable example.

2. If $(L, \leq)$ be a lattice with operation $\vee$ and $\wedge$ then for any $a, b \in L$ show that $a \leq b \Rightarrow a \wedge b = a$

3. If $u = x^y$ then show that

$$\frac{\partial^2 u}{\partial y \partial x} = \frac{\partial^2 u}{\partial x \partial y}$$

4. Define composite. Inverse functions and exponential functions.

5. Evaluate the following integral by changing the order of Integration.

$$\int_0^\infty \int_x^\infty \frac{e^{-x}}{y}\, dy\, dx.$$

10. (i) If $z = x^2 \tan^{-1}\left(\frac{y}{x}\right) - y^2 \tan^{-1}\left(\frac{y}{x}\right)$ then prove that

$$\frac{\partial^2 z}{\partial y \partial x} = \frac{x^2 - y^2}{x^2 + y^2}$$

(ii) Find all the maxima or minimum values of function $y^2 + x^2 y^2 + x^4$

11. (i) Draw the Hass diagram representing the Partial Ordering $a|b$ on

$$\delta = \langle 1, 2, 3, 4, 6, 8, 12 \rangle$$

(ii) Find sets A and B for which $|A| = 5, |B| = 6$ and $|A \cup B| = 9$; What is $|A \cap B|$?

12. (i) Find the angle of intersection of the spheres

$x^2 + y^2 + z^2 - 2x - 4y - 6z + 10 = 0$

and $x^2 + y^2 + z^2 - 6x - 2y + 2z + 2 = 0$

(ii) Show that the length of the shortest distance between the lines.

$$\frac{x-2}{2} = \frac{y+1}{3} = \frac{z}{4}\ ;$$

$2x + 3y - 5z - 6 = 0 = 3x - 2y - z + 3$ is

$97/(13\sqrt{6})$

Section–B

Note : Attempt any **two** questions. 7½×2=15

6. Prove that the lines.

$$\frac{x}{1} = \frac{y-2}{2} = \frac{z+3}{3} \text{ and } \frac{x-2}{2} = \frac{y-6}{2} = \frac{z-3}{4}$$ are coplannar also find the point of intersection.

7. Evaluate $\iint_R xy \; dx\,dy$ over the region in the possitive quadrant for which $x + y \le 1$.

8. If $f : A \to B$ and $g : B \to C$ are two bijective functions then $gof : A \to C$ is a bijective function and $(gof)^{-1} = f^{-1} o g^{-1}$

Section–C

Note : Attempt any **three** questions. 3×15=45

9. (i) Prove that dual of a complemented lattice is complemented.

(ii) If $(L, \le)$ is a lattice and $a, b, c \in L$ then

(a) $a \wedge (b \vee c) \ge (a \wedge b) \vee (a \wedge c)$

(b) $a \vee (b \vee c) \le (a \vee b) \wedge (a \vee c)$

13. (i) Evaluate

$$I = \int_0^2 \int_0^x \int_0^{x+y} e^x (y + 2z)\, dx\, dy\, dz$$

(ii) Find the Equation of the shpere which passes through the cirlce $x^2 + y^2 + z^2 - 2x + 2y + 4z - 3 = 0$, $2x + y + z - 4 = 0$ and touch the plane $3x + 4y - 14 = 0$